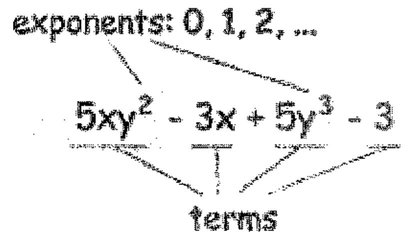
Pre-Calculus Unit 3

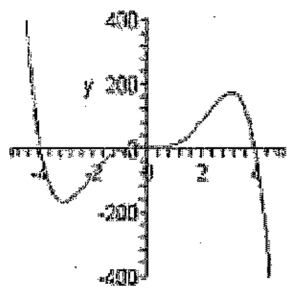
Part 1

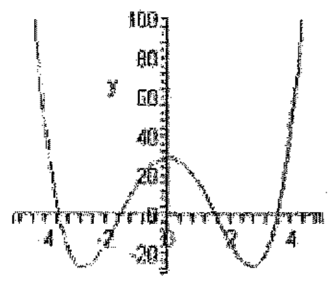
POLYNOMIALS

monomial	$\underset{1}{\underline{2x}}$
binomial	$\underset{1}{\underline{2x}} + \underset{2}{\underline{3y}}$
trinomial	$\underset{1}{\underline{2x^2}} + \underset{2}{\underline{3x}} + \underset{3}{\underline{5}}$
polynomial	$\underset{1}{\underline{3x^3}} + \underset{2}{\underline{2x^2}} - \underset{3}{\underline{6x}} + \underset{4}{\underline{2}}$



A Polynomial





Polynomial Vocabulary Review

Term: The parts of an expression that are separated by a + or – sign.

Coefficient: The number multiplied to the variable part of a term. (Ex: 13 is the coefficient of $13x^3y^2$)

Constant term (constant): A term that does not contain any variables. If there is no such term, the constant term is 0. (Ex: 7 is the constant in $13x^3 y^2+7$)

Polynomial: An expression containing one or more monomials, separated by + or – signs, with all exponents being whole numbers.

Monomial: A polynomial with one term. (Ex: $3x^3$, 5, and $2xy$ are all monomials)

Binomial: A polynomial with two terms that are not like terms. (Ex: $2x$-3 and $3x^2+4$)

Trinomial: A polynomial with three terms that are not like terms. (Ex: $3x^2+4x-5$ and $2x^3-5x+1$)

Degree of a term: In a term with one variable, the degree is the variable's exponent. In a term with more than one variable, the degree is the sum of the exponents of the variables. If the term doesn't have an exponent (is just a constant) then the degree is zero. (Ex: $-5x^3$ has degree 3 and $7x^4y^3$ has degree 7 and -4 has a degree of 0).

Degree of polynomial: The highest degree found in any term of the polynomial.

Like terms: Terms which have the same variables and corresponding powers. Like terms can be combined using addition and subtraction. Terms that are not like, cannot be combined using addition and subtraction. (Ex: $5xy^2+3xy^2$are like terms and combine to $8xy^2$)

Linear Function: A polynomial function with a degree of one.

Quadratic Function: A polynomial function with a degree of two.

Cubic Function: A polynomial function with a degree of three.

nth Degree Function: A polynomial function with a degree of n.

Name:______________________________ Date:______

1A Polynomial Graphs Unit 2 Day 1

Degree:

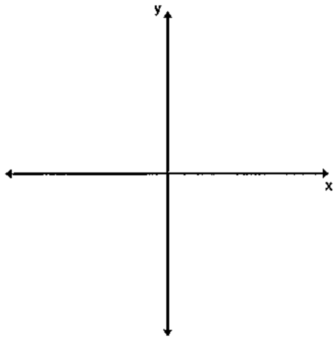
	General form of a Polynomial	Degree	$a > 0$	$a < 0$
Linear	$y = ax + b$			
Quadratic	$y = ax^2 + bx + c$			
Cubic	$y = ax^3 + bx^2 + cx + d$			
Quartic	$y = ax^4 + bx^3 + cx^2 + dx + e$			
Quintic	$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$			

What can you conclude about odd degree polynomials?

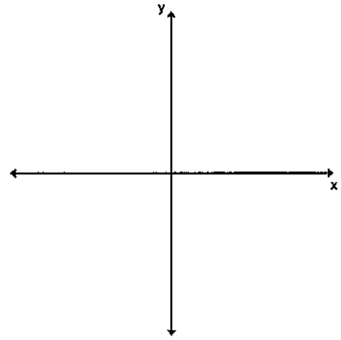
What can you conclude about even degree polynomials?

Sketch a graph of a polynomial given the following characteristics
(there are many correct answers)

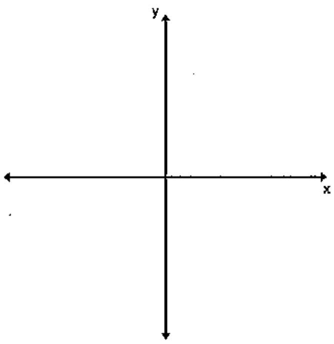
1. 4^{th} degree, negative leading coefficient, 4 real zeros



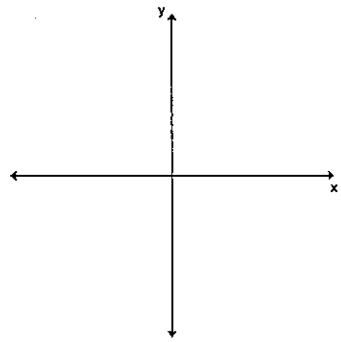
2. 3^{rd} degree, positive leading coefficient, 3 real zeros



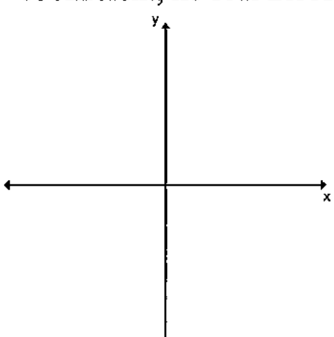
3. 5^{th} degree, negative leading coefficient, 5 real zeros



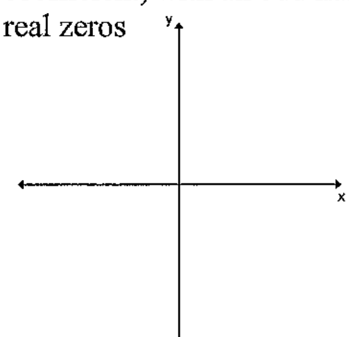
4. Even degree, positive leading coefficient, no real zeros



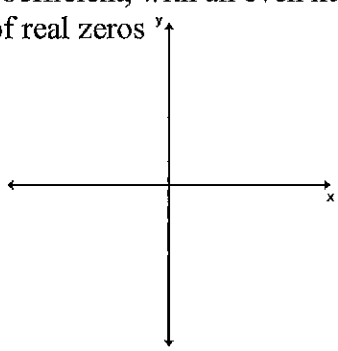
5. Odd degree, positive leading coefficient, no real zeros



6. Even degree, negative leading coefficient, with an odd number of real zeros



7. Odd degree, negative leading coefficient, with an even number of real zeros

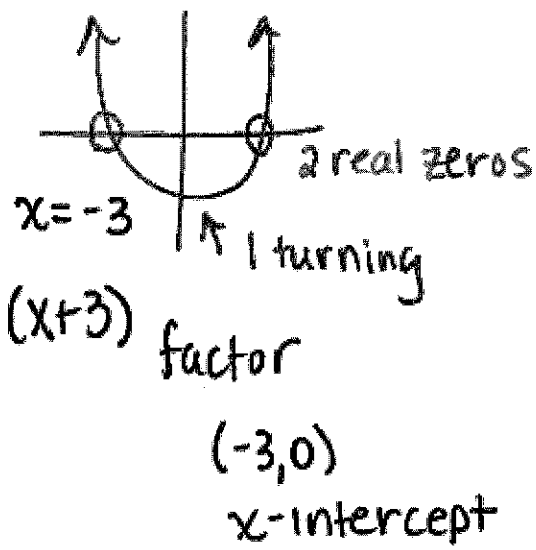


8. Which case could not be drawn?

What important feature occurred in #6 and #7?

2A Polynomials Using Factored Form to Graph Polynomials

A polynomial of degree n has at most n real zeros, and at most n-1 turning points.



Equivalent statements:

x=a is a zero of f(x)

x=a is a solution of f(x)=0

x - a is a factor of f(x)

(a, 0) is an x intercept of f(x)

$f(x) = x(x-1)^2(x+3)^3$

1. Find the degree
2. Determine if the LC is + or -
3. Use 1 and 2 to determine end behavior
4. Find the zeros and state the multiplicity
5. Sketch

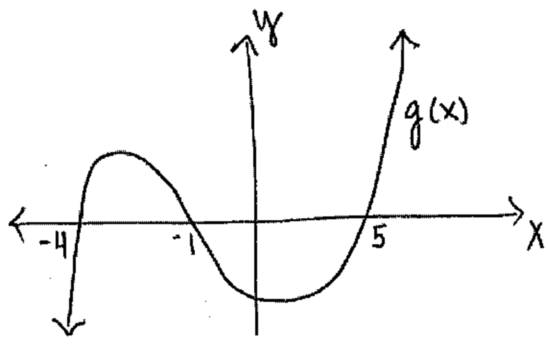
$g(x) = -(x+4)^2(x-6)^2(x+9)$

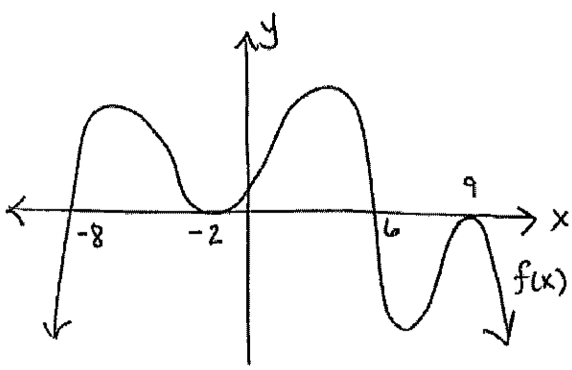
$h(x) = -\frac{1}{2}(x+4)^2(x-6)^2(x+9)^3$

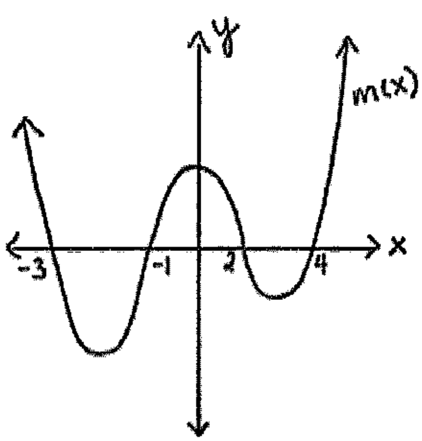
$f(x) = x^3 + 3x^2 - 4x$

$g(x) = x^4 - 26x^2 + 25$

$p(x) = x^3 - 2x^2 - 25x + 50$


y
g(x)
-4
-1
5
X


y
9
x
-8
-2
6
f(x)


y
m(x)
-3
-1
2
4
x

Precalculus Name: ____________________

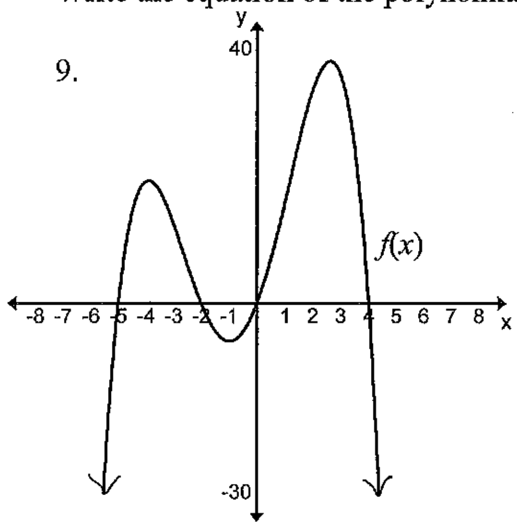
2B Graphing Polynomials Homework Mrs. O'Neill

For each of the polynomials below, state the degree, end behavior, the zeros (including multiplicity), and then draw a sketch.

1. $f(x) = -x(x-4)(x+3)^2$
2. $f(x) = 0.5(x+2)(x-5)^3(x+4)$
3. $f(x) = x(x+5)^2(x+3)$
4. $f(x) = x^2(x-1)^2(x+2)$
5. $f(x) = x^5 - 3x^4 - x^3 + 3x^2$
6. $f(x) = -x^5 - x^3 + 6x$
7. $f(x) = 2x^4 - x^3 - 6x^2$
8. $f(x) = x^6 - 25x^4 - 16x^2 + 400$

Write the equation of the polynomials sketched below.

9.



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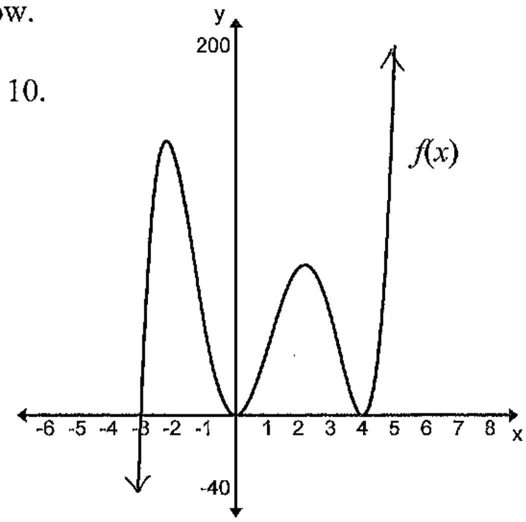

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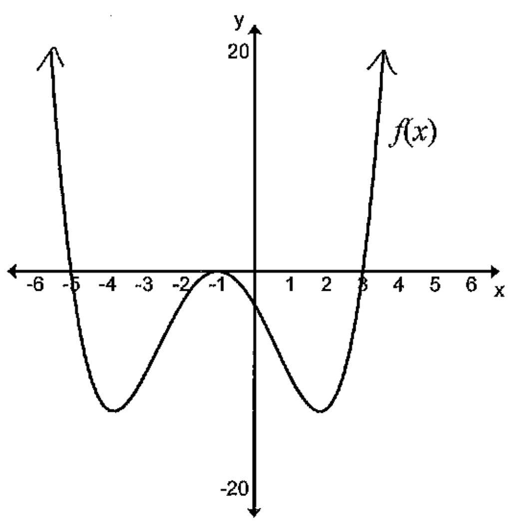

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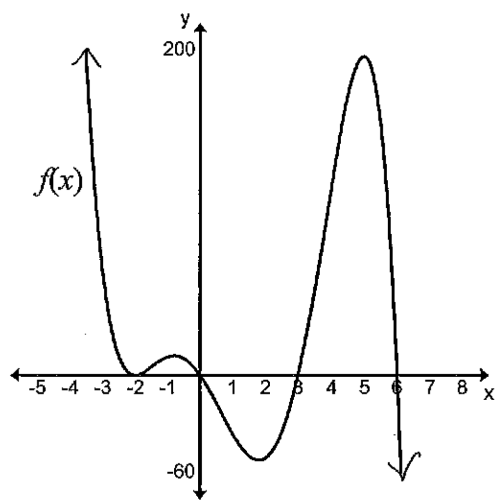

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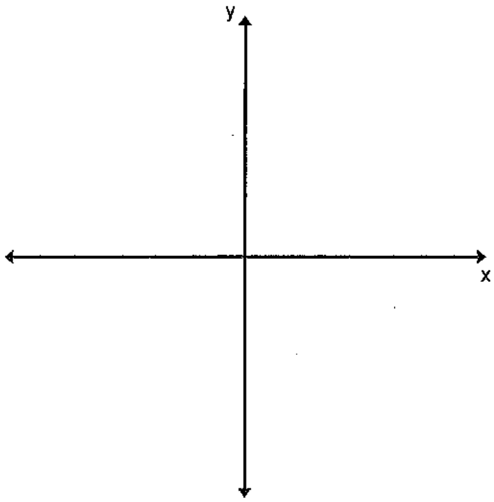
Activity 1 Name:

For each of the polynomials below, state the degree, the sign of the leading coefficient, state the zeros (including any multiplicities) and then sketch the graph.

$f(x) = -2(x-5)^3(x+1)$

Degree: LC

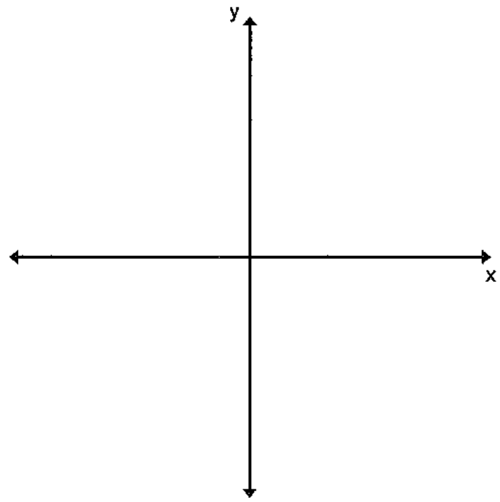
Zeros:



$f(x) = -(x+3)^2(x-2)^2(x-5)^2$

Degree: LC

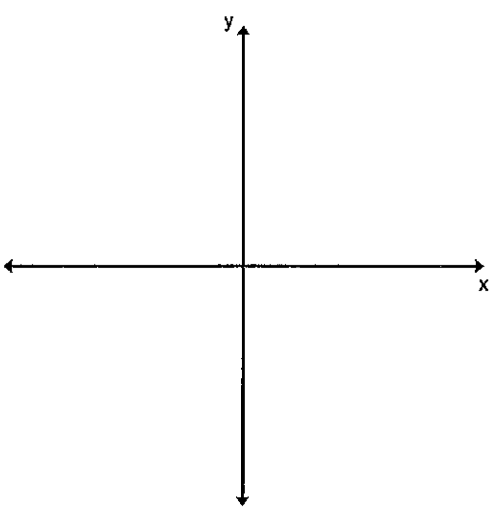
Zeros:



$f(x) = (x+1)(x-2)(x-4)$

Degree: LC

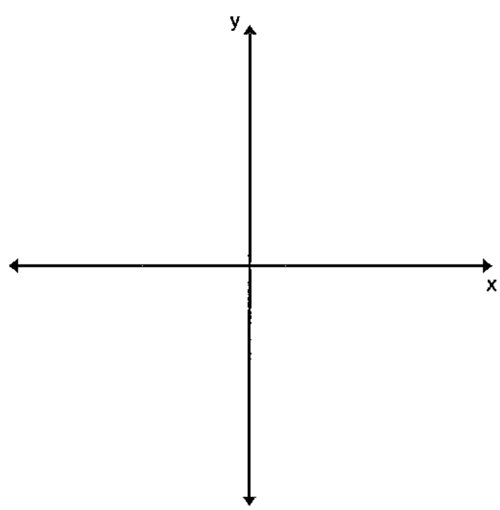
Zeros:



$f(x) = -\frac{1}{2}(x+3)(x+2)(x-1)^3$

Degree: LC

Zeros:



Activity 2 Name:

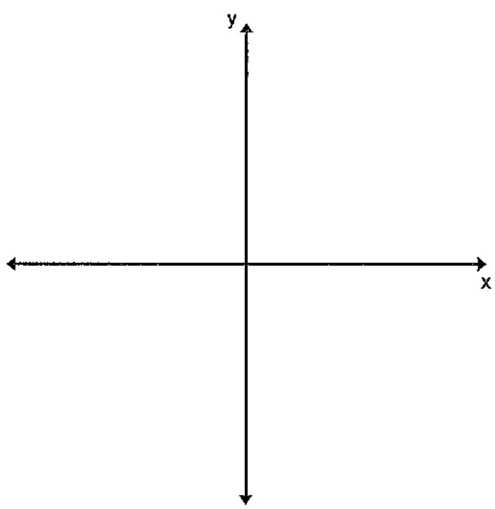
For each of the polynomials below, factor completely, state the degree, the sign of the leading coefficient, state the zeros (including any multiplicities) and then sketch the graph.

$f(x) = x^5 - 3x^4 - x^3 + 3x^2$

Factored Form:

Degree: LC

Zeros:

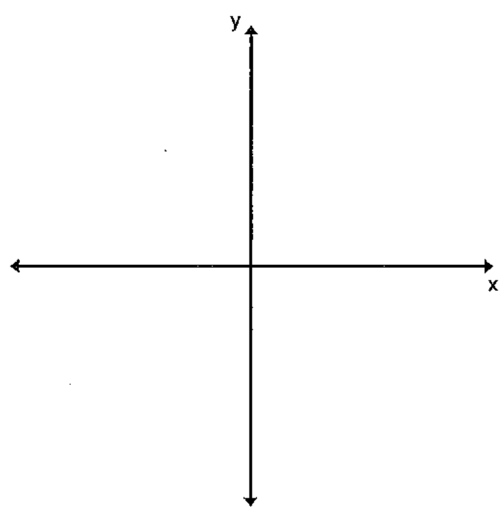


$f(x) = -x^5 + 4x^4 - 4x^3$

Factored Form:

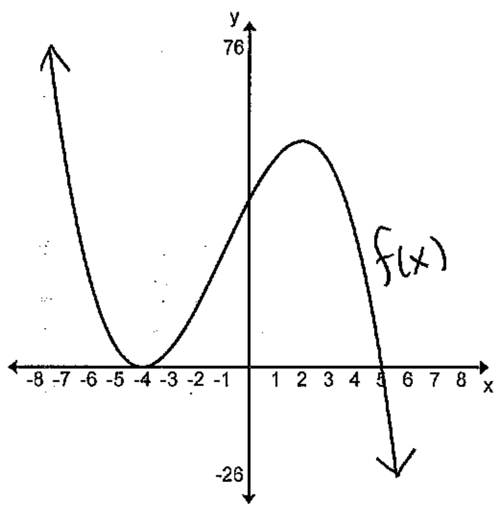
Degree: LC

Zeros:

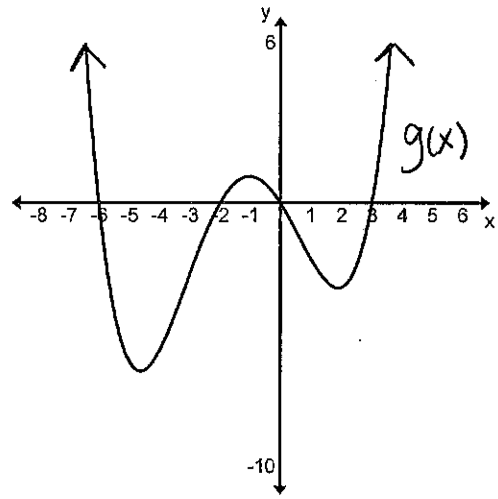


Activity 3 Name:

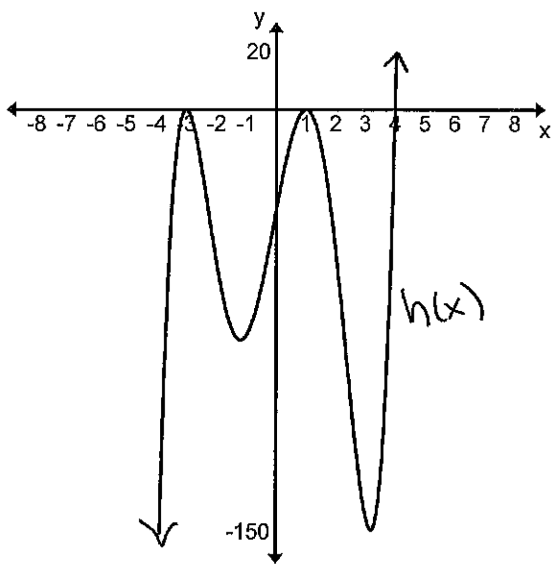
For each of the polynomials graphed below, write the equation in factored form.



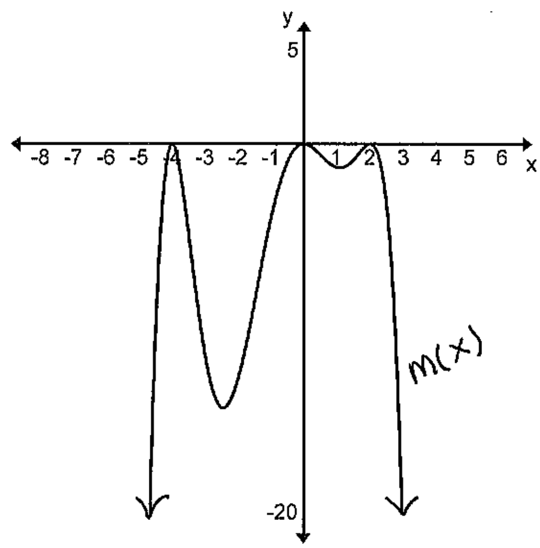
Equation: ______________________



Equation: ______________________



Equation: ______________________



Equation: ______________________

Activity 4

Name:

Completely factor each of the polynomials below.

$f(x) = x^4 + 2x^2 + 1$

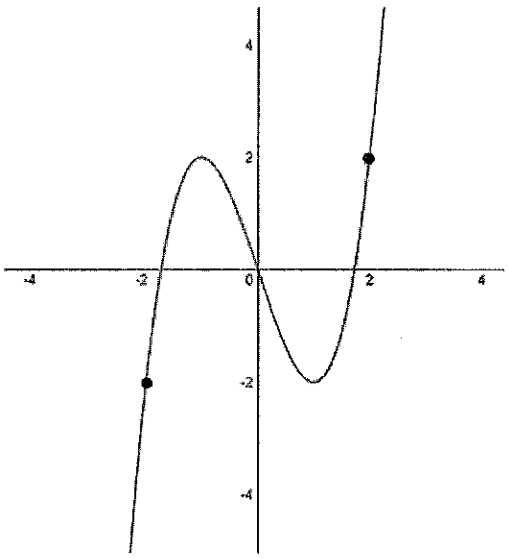
$p(x) = 25x - x^3$

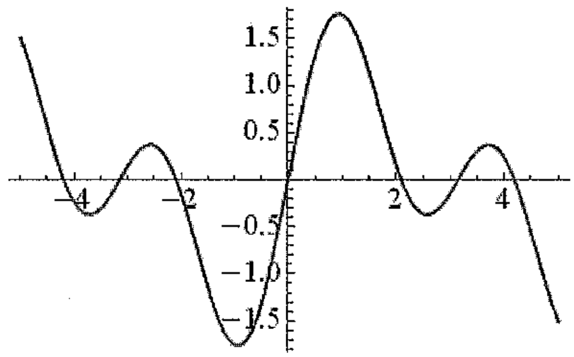
$h(x) = 2x^3 + x^2 + 2x + 1$

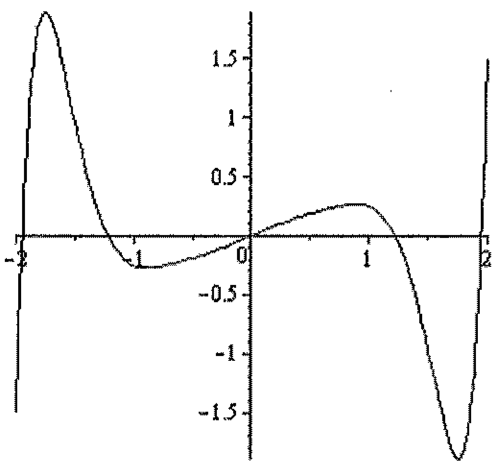
$b(x) = x^4 - 13x^2 + 36$

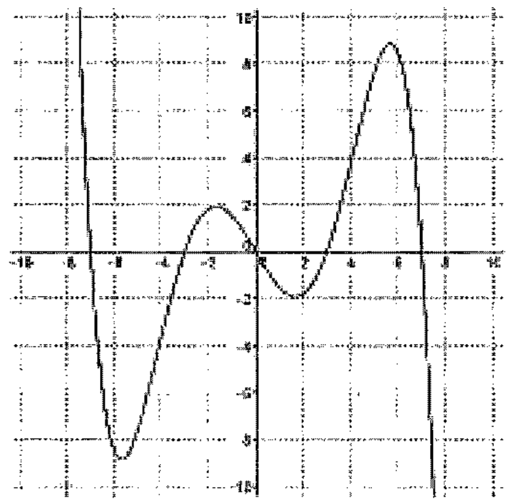
$r(x) = -x^5 + 16x^3$

Zeros of Odd Functions



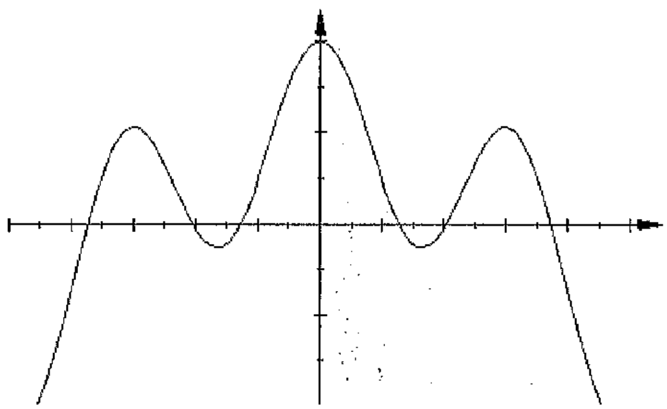


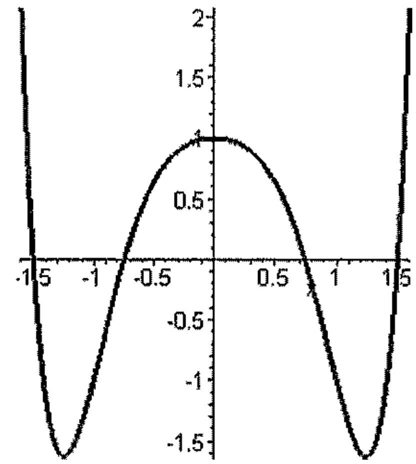


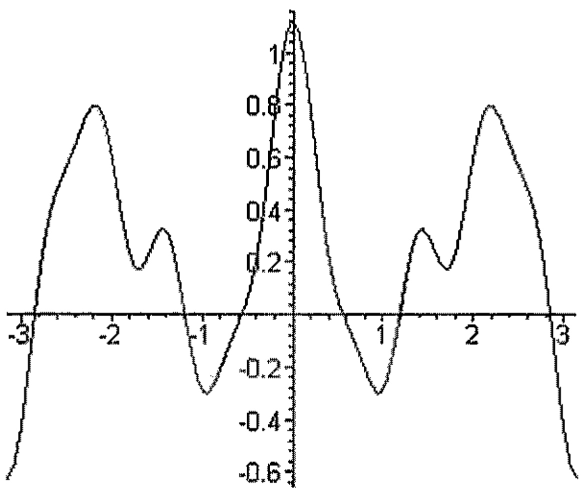


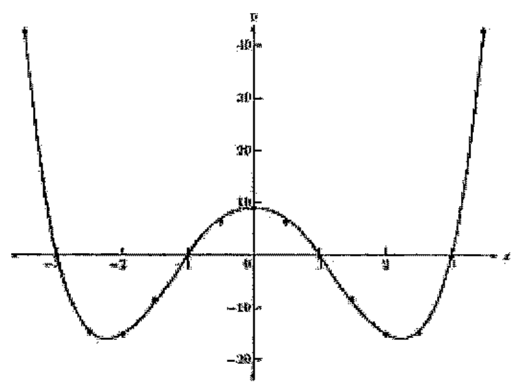
All of the functions above are odd (symmetric about the origin). What common features do their zeros share?

Zeros of Even Functions









All of the functions above are even (symmetric about the y-axis). What common features do their zeros share?

Write a polynomial based on the given set of conditions below first in factored form and then in standard form.

1. An even function with zeros 3 and 6.

2. An odd function with zeros 2 and 4.

3. An odd function with zeros 1, 3 and $\sqrt{2}$.

This lesson assumes that you know how to graph a polynomial and find the zeros.

Solve graphically and analytically $x^2 - 10 < 3x$

1. Get a zero on the right side of the inequality.

2. If possible, factor.

3. Determine where the polynomial is = 0. Why? When the value of a polynomial changes sign (moves from + to - or from - to +) the graph must go through ZERO.

Sketch the graph:

Analyze the graph and write your answer in interval notation.

Example 2: Solve $x^3 - 2x^2 - 3x > 0$

Example 3: Solve $x^3 + 3x^2 + x + 3 \leq 0$

Example 4: Solve $(x-4)^2(x^2-1) > 0$

Example 5: Solve $(x+8)^2(x+5)(x+7)^2 \geq 0$

Example 6:

Write a polynomial inequality with the solution: $\{-1\} \cup \{2\} \cup [3,\infty)$

Example 7:

Create a polynomial inequality with the following solution set. Justify your answer.

$(-\infty,-2)\cup(-1,0)\cup(0,\infty)$

Homework: Solve.

1. $x^3 < x$

2. $x^3 + x^2 - 6x > 0$

3. $x^3 > x^2$

4. $x^4 + x^3 < 4(x+4)$

5. $2x^2 + 5x - 7 \leq 0$

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Name________________________________

Polynomial Inequalities

Date________________ Period____

Solve each inequality. Write your answer using interval notation

1) $(x-4)(x+3)<0$

2) $(x-4)(x+1)\geq 0$

3) $(x-1)(3x-4)\geq 0$

4) $(x+8)(x+2)(x-3)\geq 0$

5) $x^2+5x+4\leq 0$

6) $x^2-14x+49\geq 0$

7) $x^2-4x-32>0$

8) $x^2+16x+24>6x$

9) $(x+5)(x-2)(x-1)(x+1)<0$

10) $(x+8)^2(x+5)(x+7)^2\geq 0$

Critical thinking question:

11) Write a polynomial inequality with the solution: $\{-1\}\cup\{2\}\cup[3,\infty)$

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Polynomial Inequalities

Date________________ Period____

Solve each inequality.

1) $(x-4)(x+3)<0$

$(-3, 4)$

2) $(x-4)(x+1)\geq 0$

$(-\infty, -1]\cup[4, \infty)$

3) $(x-1)(3x-4)\geq 0$

$(-\infty, 1]\cup\left[\frac{4}{3}, \infty\right)$

4) $(x+8)(x+2)(x-3)\geq 0$

$[-8, -2]\cup[3, \infty)$

5) $x^2+5x+4\leq 0$

$[-4, -1]$

6) $x^2-14x+49\geq 0$

$(-\infty, \infty)$

7) $x^2-4x-32>0$

$(-\infty, -4)\cup(8, \infty)$

8) $x^2+16x+24>6x$

$(-\infty, -6)\cup(-4, \infty)$

9) $(x+5)(x-2)(x-1)(x+1)<0$

$(-5, -1)\cup(1, 2)$

10) $(x+8)^2(x+5)(x+7)^2\geq 0$

$\{-8\}\cup\{-7\}\cup[-5, \infty)$

Critical thinking question:

11) Write a polynomial inequality with the solution: $\{-1\}\cup\{2\}\cup[3,\infty)$

Example: $(x+1)^2\cdot(x-2)^2(x-3)\geq 0$