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Modeling a High Road/Low Road race quantitatively using Desmos

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The High Road/Low Road race is often used as a qualitative prediction activity. A marble is released simultaneously on two elevated tracks that differ only in the middle section where the Low Road dips before finishing at the same height as the High Road. Typically a minority of students correctly predict that the Low Road will finish first.[1] In this investigation an energy and segmented constant acceleration-based Desmos model[2] is developed to predict the time required to traverse each track. The effect of rotational inertia on the model is also explored. Those predictions are then compared with the time measured by photogates and video analysis.

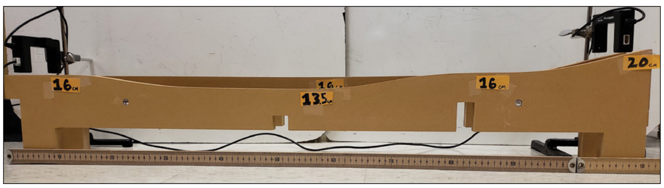

Fig. 1. High Road/Low Road Apparatus.

Analyzing the situation

Since the marbles start at rest from the same height, energy conservation suggests that the marbles should share the same speed everywhere they have equal height. The marble travels faster on the lower middle section of the Low Road than the High Road. Quantifying the total time prediction is complicated by nonconstant acceleration. Each track can be broken down into a series of straight line segments. Energy conservation can be used to determine the initial and final speeds for each segment. Treating each segment like constant acceleration motion, the time to traverse each segment can be approximated as the segment length divided by the average speed. The total finish time can be computed from the sum of the segment times.

Constructing a Desmos model

Desmos is a powerful free HTML5 graphing calculator. The equations to implement our model rely on a series of x and y positions. An image of the apparatus is uploaded to Desmos and resized until the scale in the photo matches the axes. Twenty points are dragged against the background image to specify the end points of 19 line segments along the path.

With these coordinates now specified, the change in gravitational potential energy of each location from the release point is computed using

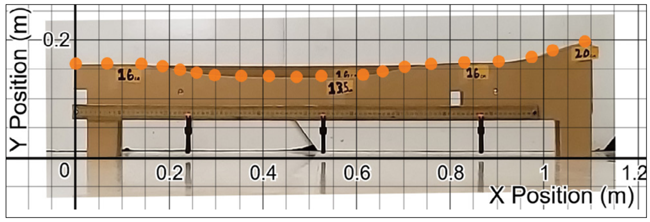

Fig. 2. Desmos model – Mapping coordinates onto image.

$$\Delta U_g = mg(y - y_0). \quad (1)$$

If the marble rolls without slipping, the gravitational potential energy can be set to equal to the kinetic energy,

$$\Delta U_g = -\Delta K, \quad (2)$$

including translational and rotational kinetic energy,

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}fmr^2\tfrac{v^2}{r^2} \quad (3)$$
$$K = (1+f)\cdot\tfrac{1}{2}mv^2,$$

where f is the rotational inertia fraction for a rotating object with a circular profile of radius r as viewed along the axis of rotation. For a solid sphere, $f = 2/5$. Once the velocity at each location is known, the time for each segment of motion is computed by

$$t_{\text{seg}} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{\bar{v}}. \quad (4)$$

Model vs. experiment

As summarized in Table I below, the Desmos model predicts a total time of 1.74 s for the High Road and 1.63 s for the Low Road. The actual experiment was repeated 50 times, yielding an average photogate time of 1.870 +/- 0.005 s for the High Road, and 1.755 +/- 0.012 s for the Low Road. Video analysis using Tracker[3] yielded 1.86 s and 1.77 s. When rotational inertia is ignored and f is set to zero, the model yields times much shorter than experiment.

Table I. Desmos model and experimental results.

	Model		Experiment	
	f = 2/5	f = 0	Photogate	Video
High	1.74 s	1.47 s	1.870 s	1.86 s
Low	1.63 s	1.37 s	1.755 s	1.77 s
H - L	0.11 s	0.10 s	0.115 s	0.09 s

While photogate and video time measurements were within 0.02 s, the model was about 7% shorter than experimental measurements. Video was collected at 240 fps and analyzed using Tracker.

It is interesting to compare the speed vs. horizontal position predicted by the model shown in Fig. 3 with the video analysis shown in Fig. 4. Instead of maintaining speed, the High Road appears to slow down on the flat section. There may be some energy dissipation due to friction, but perspective error may play a bigger role. Figure 4 gives a shorter apparent length for the High Road, even though both tracks are 110 cm.

The picture in this Desmos model can be readily swapped for another image and the 20 points repositioned. This Desmos, raw video, and Tracker file can be found at Ref. 4, along with a link to numerous Desmos models and simulations I have developed.

References

1. W. Leonard and W. Gerace, "The power of simple reasoning," *Phys. Teach.* **34**, 280 (May 1996).
2. See https://www.desmos.com/calculator/jolaxosv2o.
3. "Tracker video analysis and modeling tool," https://physlets.org/tracker/.
4. Supplementary files and Desmos simulations by Dan Hosey: https://www.mrhosey.com/desmos.

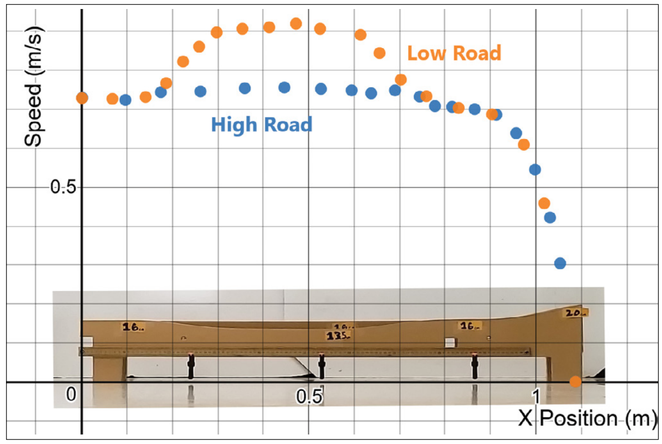

Fig. 3. Desmos model speed vs horizontal position.

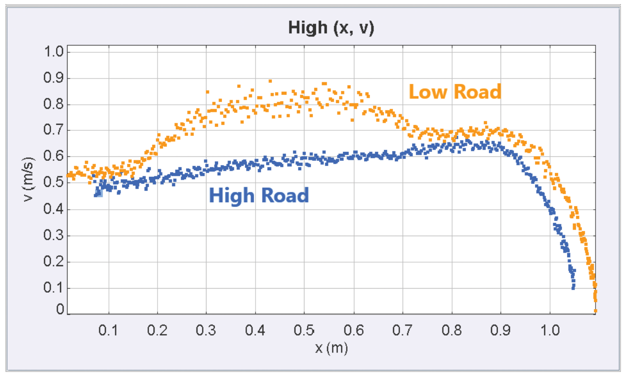

Fig. 4. Video analysis of speed vs. horizontal position.